# FINAL RESULTS OF A NEW TEST OF RELATIVITY

Timothy P. Krisher, Lute Maleki, Lori E. Primas, George F. Lutes, *
Ronald T. Logan, and John D. Anderson
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

and

Clifford M. Will
McDonnell Center for the Space Sciences
Department of Physics, Washington University
St. Louis, Missouri

### Abstract

**We report on the final results of an analysis of data obtained in a recent experiment to test the isotropy of the one-way velocity of light using instrumentation of the Deep Space Network (DSN). The data consisted of measurements of the relative phase versus time of the 100 MHz output of two hydrogen maser frequency standards separated by 29 kilometers which were compared by propagating signals one-way over an ultra-stable fiberoptics cable. Uninterrupted phase records spanning nearly five days were generated simultaneously at each site, thereby providing us with the capability to subtract out errors due to either a frequency offset between the masers or a temperature-dependent delay along the fiberoptics cable. Adding the records removed the maser frequency offset error, while differencing the records removed the cable delay error. It was not possible to subtract out both errors simultaneously.**

**By taking 1000 second samples and low-pass filtering the phase records for periodicities of 12 hours or longer, we observed daily phase variations of less than 20 degrees in the differenced data and of less than 1 degree in the added data. Both spectral analysis and linear regression techniques were used to infer from the phase records limits on two possible anisotropic variations in the one-way velocity of light which could arise from the motion of the earth with respect to the cosmic microwave background. The differenced record permits a limit to be set on an anisotropy which is linear in this velocity, while the added record can be used to limit a quadratic dependence. A theoretical interpretation of the resulting limits is given in terms of the test theory of relativity proposed by Mansouri and Sexl.**

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# I. INTRODUCTION

At the last PTTI meeting we reported on the results of a preliminary analysis of data recently obtained in an experiment to test the isotropy of the one–way velocity of light[1]. A more complete analysis of the data that has been performed since then is the subject of this report. The data consists of measurements of the relative phase versus time of two hydrogen maser frequency standards separated by a baseline of 21 kilometers that were compared by propagating the 100 Mhz output signals of the masers over a highly stable fiber optics link. Because the phase measurements were made continuously over several days, the data can be used to test for a possible variation in the velocity of light that could arise from the motion of the Earth through a cosmological reference frame. Although daily phase variations were apparent in the data, it is not possible to claim to have detected such an effect because of other possible causes. Nevertheless, we have been able to infer from the data an interesting limit on the size of this effect.

This report is organized as follows. A brief review of the most important features of the instrumentation and procedures which were used in the experiment is given in Section II (see Reference [1] for further details). In Section III it is shown how the data can be used to limit the size of $\delta c/c$. An interpretation of this limit within the "test theory" of relativity of Mansouri and Sexl is presented in Section IV. Concluding remarks are given in Section V.

# II. REVIEW OF INSTRUMENTATION AND PROCEDURE

The experiment was conducted at the Deep Space Network (DSN) Deep Space Communications Complex located in the Mojave desert at Goldstone, California during November of 1988. Two hydrogen masers were located, respectively, at Deep Space Station 13 (DSS 13) and DSS 14, which are separated by a baseline of 21 kilometers. A 29 kilometer fiber optic link that is buried five feet underground is used to propagate the 100 Mhz output signals between the masers. Relative phase measurements were performed once per minute using Hewlett–Packard 8753A Network Analyzers located at each station. IBM personal computers were used to store the measurements onto 3.5 inch micro disks. Dual phase measurements were performed simultaneously at each station. In addition, it was possible to propagate both maser signals along a single optical fiber. Thus dual phase records were generated which could later be added or subtracted for the purpose of calibrating the data.

# III. ANALYSIS

For a possible violation of the theory of special relativity, there could occur an observed variation in the velocity of light of the form

$$\delta c/c_0 = c_1 \cos\theta + c_2 \cos^2\theta, \tag{1}$$

where $\theta$ is the angle between the propagation path and a preferred direction in space. This direction will be taken to be along the velocity vector of the Earth through the cosmic microwave background, which has been determined by observations of the dipole anisotropy of the background[2]. The coefficients $c_1$ and $c_2$ are dimensionless, both having the value of exactly zero in special relativity, while $c_0$ represents a fiducial value for the speed of light (nominally equal to $3 \times 10^5$ km/sec). For convenience, we will define $c_0 \equiv 1$. Because the measured phase difference between the two masers is related to $c$ by the definition $\phi = 2\pi\nu L/c$, where $\nu = 100$ MHz and $L = 21$ km, equation (1) yields

$$\delta\phi/\phi_0 = \phi_1 \cos\theta + \phi_2 \cos^2\theta, \tag{2}$$

where $\phi_1 = -c_1$ and $\phi_2 = c_1^2 - c_2$.

The two limiting sources of error in the experiment are 1) the maser stability, and 2) the temperature dependence of the fiber optics link. By differencing the dual phase records, the link error can be eliminated, yielding

$$(\delta\phi/\phi)_{DSS\ 13} - (\delta\phi/\phi)_{DSS\ 14} = 2\phi_1 \cos\theta + \text{(maser error)}, \tag{3}$$

while by adding the phase records the maser error can be eliminated, so that

$$(\delta\phi/\phi)_{DSS\ 13} + (\delta\phi/\phi)_{DSS|14} = 2\phi_2 \cos^2\theta + \text{(link error)}. \tag{4}$$

These differenced and added phase records are shown in Figures 1 and 2, respectively. In order to remove unwanted high frequency noise from the data, the records have been low–pass filtered for periodicities of 12 hours or longer[3]. In addition, the data has been sampled at 1000 second intervals in order to take advantage of the high frequency stability of the hydrogen maser at this integration time and also to reduce the size of the data records. It was necessary to truncate the differenced data after 72 hours because of an anomalously large phase excursion that occurred.

Daily phase variations are clearly apparent in these plots. In the case of the differenced data these variations could be due to the effects on the masers of changes in barometric pressure (see, for example, Reference [4]), while the smaller variations seen in the added data are consistent with temperature–dependent variations in the delay of the fiber optics link. The size of these systematic errors determines the limits which can be set for the anisotropy coefficients $c_1$ and $c_2$. In order to determine these limits, equations (3) and (4) have been evaluated for the particular geometry of the experiment at the times of the phase measurements (see Figure 3). From least–squares fits of each equation to the corresponding phase record, there resulted the limits

$$c_1 < 3.5 \times 10^{-7}, \tag{5a}$$

$$c_2 < 2 \times 10^{-8}. \tag{5b}$$

The limit in equation (5a) resulted from a simple fit of the predicted cosine variation to the differenced data, where the number quoted is the fitted amplitude. Equation (5b) resulted from modeling in addition to the predicted cosine–squared variation a twenty–four hour cosine of arbitrary amplitude and phase to account for a possible diurnal variation in the link delay. In the next section the meaning of these limits will be discussed.

## IV. THEORY

Great care must be taken in determining the theoretical consequences of these experimentally derived limits. Their expected values for a possible violation of special relativity can be determined only by a detailed application of the theory in question to the particular instrumentation and procedures which were used in the experiment. A complete dynamical theory would account for all physical forces which could arise from motion through a preferred reference frame. This kind of a framework has been adopted by Will and Haugan, for example, in analyzing the outcomes of relativity experiments[5]. In our experiment, these forces could directly affect the hyperfine transition of the hydrogen atoms in the maser or perhaps change the length of the fiber optics link. A more simplified approach to understand the possible consequences of the experiment is provided by the formalism of Mansouri and Sexl, in which only the kinematics of the motion is considered[6]. We will adopt this approach and assume that it is consistent with the actual dynamics that could be involved. Whether there exists a more interesting interpretation of the experiment remains an open question for the present.

In the formalism of Mansouri and Sexl, the Lorentz transformations of special relativity are parameterized according to:

$$t = aT + \vec{\epsilon}\cdot\vec{x}, \tag{6a}$$

$$\vec{x} = d\vec{X} - b\vec{v}T + \frac{b-d}{v^2}\vec{v}(\vec{v}\cdot\vec{x}), \tag{6b}$$

where $(t, \vec{x})$ are the space–time coordinates for a frame moving with a velocity $\vec{v}$ with respect to the preferred frame $(T, \vec{X})$. In special relativity the parameters $(a,\ b,\ d,\ \vec{\epsilon})$ have the values of $a = b^{-1} = (1-v^2)^{1/2}$, $d = 1$, and $\vec{\epsilon} = -\vec{v}$. For light propagating isotropically with respect to the preferred frame, these transformation equations predict a relative variation in the phases of two frequency standards in the moving frame given by [7]

$$\delta\phi/\phi_0 = v(1+2\alpha)\cos\theta + v^2(\frac{1}{2}+\delta-\beta)\cos^2\theta, \tag{7}$$

where $\theta$ is the angle of propagation with respect to $\vec{v}$ and the parameters $(\alpha,\ \beta,\ \delta)$ result from the expansion of $(a,\ b,\ d)$, respectively, in powers of $v^2$. In special relativity, their values are $\alpha = -1/2$, $\beta = 1/2$, and $\delta = 0$. Because the phase is related to the speed of light $c$ according to the definition $\phi = 2\pi\nu L/c$, the observed values of the anisotropy coefficients $c_1$ and $c_2$ are:

$$c_1 = -(1+2\alpha)v, \tag{8a}$$

$$c_2 = [(1+2\alpha)^2 - (\frac{1}{2}+\delta-\beta)]v^2. \tag{8b}$$

Taking $v = 10^{-3}$ (300 km/sec) and comparing equations (5a) and (8a), the "time-dilation" parameter $\alpha$ is thus seen to be limited by

$$\mid \alpha + \frac{1}{2} \mid < 1.75\times10^{-4}. \tag{9}$$

The parameters ($\beta$, $\delta$) are not significantly constrained because of the small value of $v^2/c_0^2$.

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# V. CONCLUSIONS

By exploiting the existing time and frequency instrumentation of the Deep Space Network we have been able to limit a fundamental variation in the one–way velocity of light to a level of $\delta c/c < 3.5 \times 10^{-7}$. This result is compared to the results of related tests in Figure 4. The GPS common–view time–transfer method has the capability to provide a precise test over a long baseline. Accuracies of 10 ns which have been obtained over intercontinental baselines implies the limit given in Figure 4[8], assuming that there are no significant correlations with spacecraft position error. A rigorous analysis of the GPS data could verify this limit, which is seen to be comparable to our result. For comparison, the results of two precise laboratory experiments have also been quoted in Figure 4. While these two limits appear to be more stringent, they are possibly not because the light propagation delay (or its variation) was not directly measurable in the experiments[9,10]. Instead, the limits were inferred from frequency comparisons. Furthermore, the propagation paths were limited to short baselines defined by the size of the apparatus. These possible shortcomings were avoided in our experiment by the direct comparison of the maser signal phases over a long baseline.

The equipment and procedures which were used in the experiment did not exceed operational DSN requirements. Several improvements could be made in the future for the special purposes of this experiment. In particular, the maser environments could be controlled and monitored much more stringently, which could lead to a reduction of the limiting phase variations seen in Figures 1 and 2.

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low–pass filtering. It should also be noted that Figure 6 of Reference [1] corresponds to adding the data records and not differencing them, as was mistakenly stated in the text.

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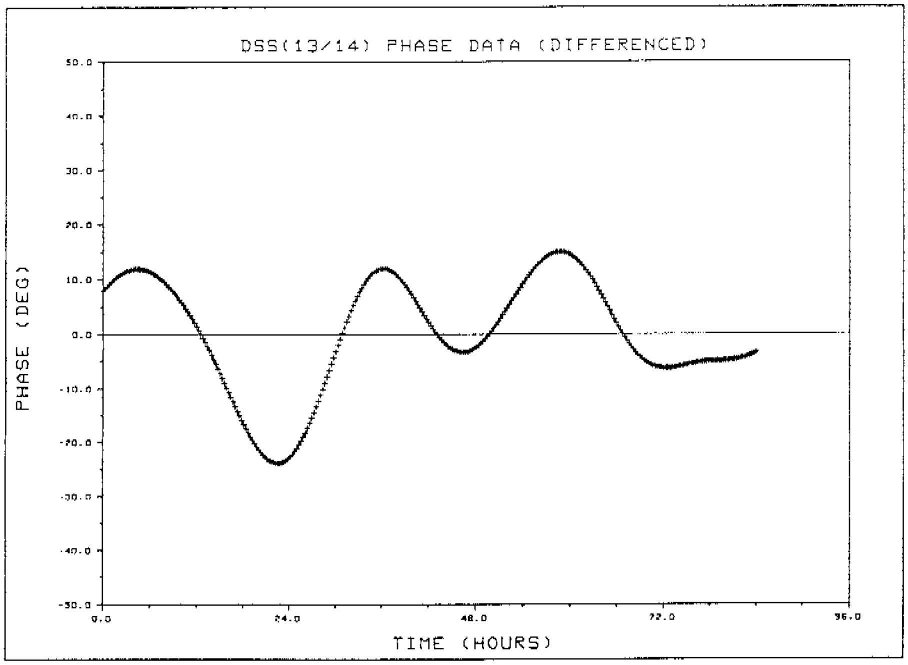


Figure 1.

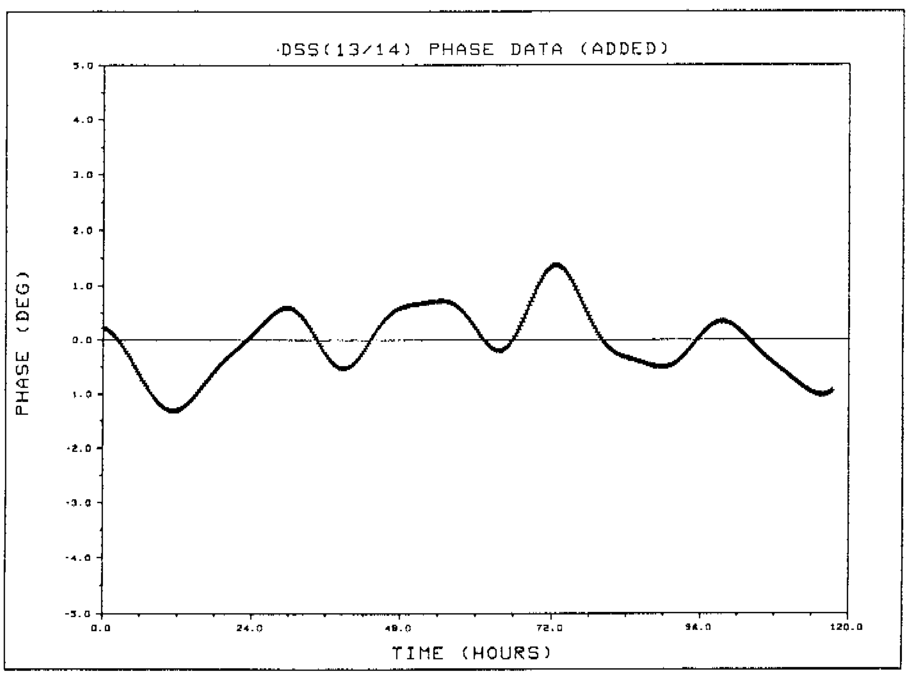


Figure 2.

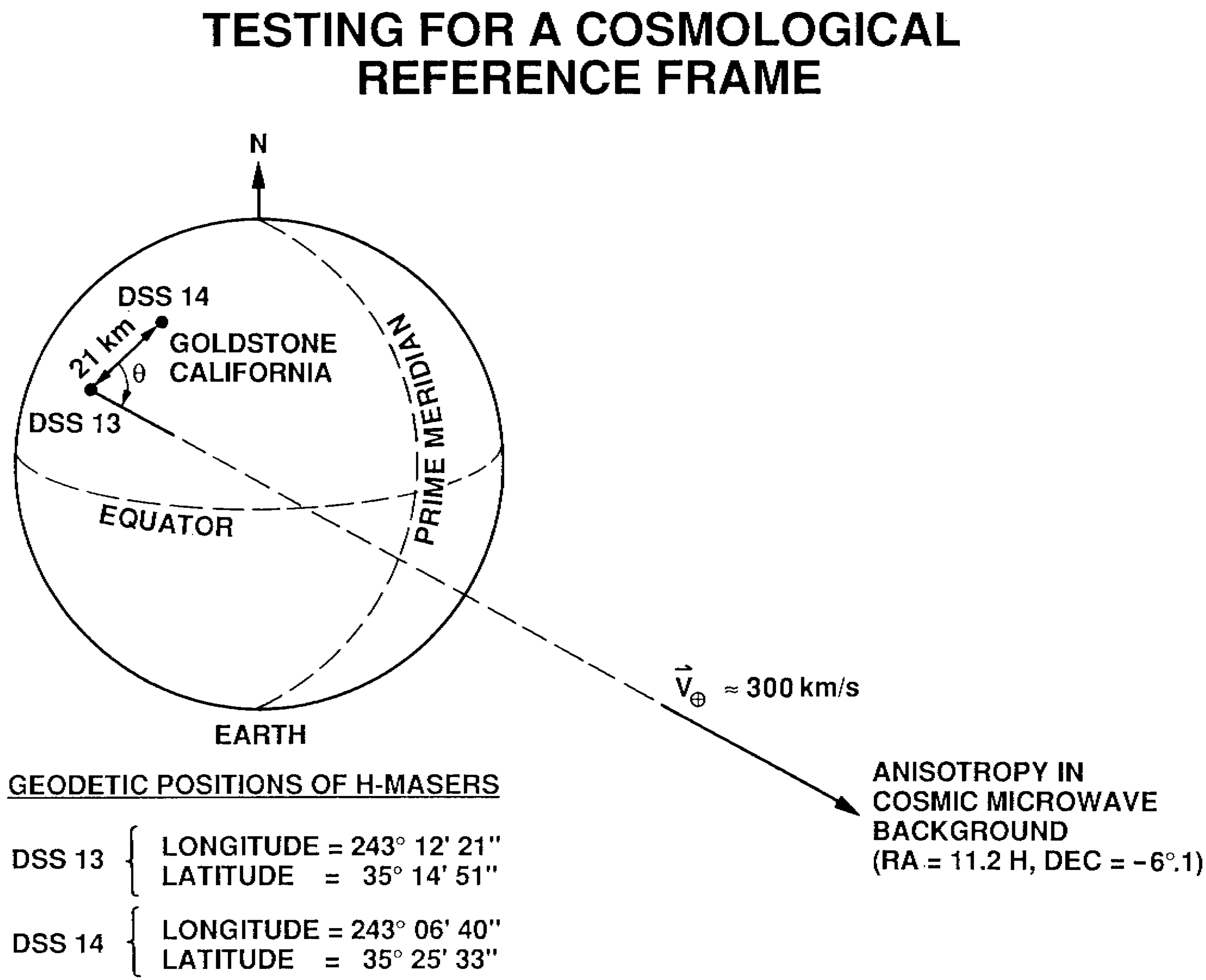
TESTING FOR A COSMOLOGICAL
REFERENCE FRAME
N
DSS 14
21 km
GOLDSTONE
CALIFORNIA
θ
DSS 13
PRIME MERIDIAN
EQUATOR
EARTH
$\vec{V}_{\oplus}$ ≈ 300 km/s
ANISOTROPY IN
COSMIC MICROWAVE
BACKGROUND
(RA = 11.2 H, DEC = −6°.1)
GEODETIC POSITIONS OF H-MASERS
DSS 13 LONGITUDE = 243° 12' 21"
LATITUDE = 35° 14' 51"
DSS 14 LONGITUDE = 243° 06' 40"
LATITUDE = 35° 25' 33"

Figure 3.

# TESTS OF THE ISOTROPY OF THE ONE-WAY VELOCITY OF LIGHT (ca 1989)

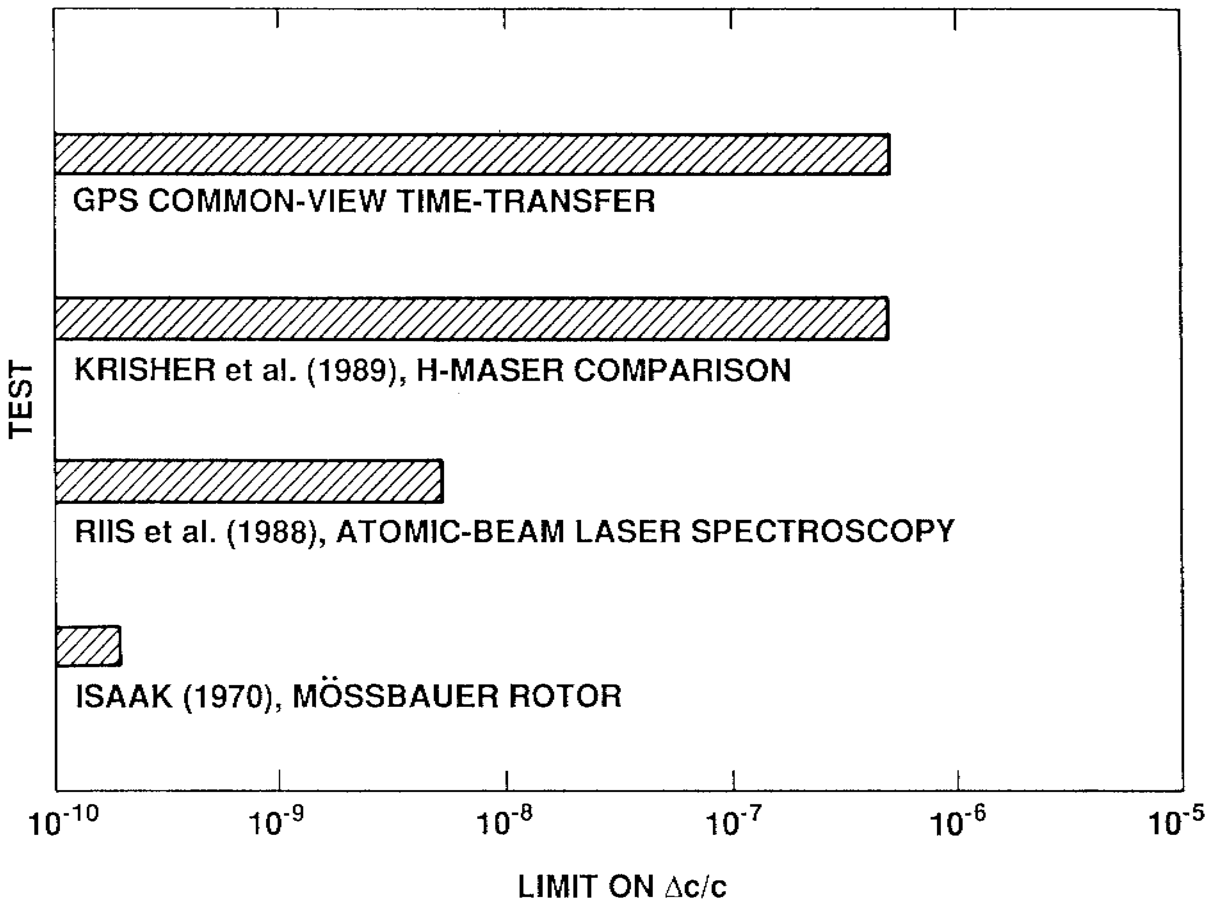


Figure 4.